

The many scales to cosmic homogeneity: Use of multiple tracers from the SDSS

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ABSTRACT

We carry out multifractal analyses of multiple tracers namely the main galaxy sample, the LRG sample and the quasar sample from the SDSS to test the assumption of cosmic homogeneity and identify the scale of transition to homogeneity, if any. We consider the behaviour of the scaled number counts and the scaling relations of different moments of the galaxy number counts in spheres of varying radius R to calculate the spectrum of the Minkowski-Bouligand general dimension $D_q(R)$ for $-4 \leq q \leq 4$. The present analysis provides us the opportunity to study the spectrum of the generalized dimension $D_q(R)$ for multiple tracers of the cosmic density field over a wide range of length scales and allows us to confidently test the validity of the assumption of cosmic homogeneity. Our analysis indicates that the SDSS main galaxy sample is homogeneous on a length scales of $80 h^{-1}$ Mpc and beyond whereas the SDSS quasar sample and the SDSS LRG sample show transition to homogeneity on an even larger length scales at $\sim 150 h^{-1}$ Mpc and $\sim 230 h^{-1}$ Mpc respectively. These differences in the scale of homogeneity arise due to the effective mass and redshift scales probed by the different tracers in a Universe where structures form hierarchically. Our results reaffirm the validity of cosmic homogeneity on large scales irrespective of the tracers used and strengthens the foundations of the Standard Model of Cosmology.

Key words: methods: data analysis - galaxies: statistics - large-scale structure of Universe

1 INTRODUCTION

The Cosmological Principle is one of the most fundamental assumption in modern cosmology which states that the Universe is statistically homogeneous and isotropic on sufficiently large scales. Homogeneity implies translational invariance i.e. the Universe looks same to all observers at different locations. Isotropy implies rotational invariance i.e. the Universe looks the same in all directions. These two assumptions play an important role in the analysis and interpretation of various cosmological observations. One can not prove these assumptions in a rigorously mathematical sense but it is possible to verify them using various observations. The near uniform temperature of the cosmic microwave background radiation is considered to be one of the best

possible evidence in favour of isotropy (Penzias & Wilson 1965; Smoot et al. 1992; Fixsen et al. 1996). There are many other observations in favour of isotropy such as the angular distributions of radio sources (Wilson & Penzias 1967; Blake & Wall 2002), the X-ray background (Peebles 1993; Wu et al. 1999; Scharf et al. 2000), the Gamma-ray bursts (Meegan et al. 1992; Briggs et al. 1996), the distribution of galaxies (Marinoni et al. 2012; Alonso et al. 2015), the distribution of supernovae (Gupta & Saini 2010; Lin et al. 2015) and the distribution of neutral hydrogen (Hazra & Shafieloo 2015). But isotropy around us alone does not guarantee homogeneity of the Universe because homogeneity and isotropy may or may not coexist. The Universe can be isotropic around a point without being homogeneous. The homogeneity can be inferred from isotropy only when there is isotropy around each and every point in the Universe. So it is not quite straightforward to infer homogeneity of the Universe from isotropy around us alone.

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The statistical properties of galaxy distributions can be characterized by the correlation functions (Peebles 1980). The two point correlation function quantifies the clustering strength which is well described by a power law on small scales and vanishes on large scales conforming to large scale homogeneity. However a major drawback of the correlation function for the analysis of homogeneity arise from the fact that it assumes a mean density on the scale of the survey which is not a defined quantity below the scale of homogeneity. It is well known that the galaxy distribution behave like a fractal on small scales. But Pietronero (1987) analyzed the CfA1 survey and suggested that the distribution of galaxies is fractal to arbitrary large-scales. Further, Coleman & Pietronero (1992) supported the argument of Fractal Universe by analyzing different samples. There are other studies which claim the absence of any transition to homogeneity out to scale of the survey (Amendola & Palladino 1999; Joyce et al. 1999; Sylos Labini et al. 2007, 2009a,b,c; Sylos Labini 2011b; Park et al. 2016). Borgani (1995) claimed that the fractal structure is valid at small scale but at large-scale the Universe is Homogeneous. Guzzo (1997) analyzed the Perseus-Pisces redshift survey and claimed that galaxies are clustered at small scale and intermediate scale but is homogeneous on large-scale. Cappi et al. (1998) found the volume-limited subsample of SSRS2 to be consistent with both fractality at small scale and homogeneity on large-scale. Bharadwaj et al. (1999) find homogeneity at scale larger than $80 h^{-1}$ Mpc, using Multifractal analysis of the LCRS sample. Pan & Coles (2000) used fractal analysis to PSCz and find the homogeneity scale even at $30 h^{-1}$ Mpc. Yadav et al. (2005) have analyzed the two-dimensional strips from SDSS DR1 using Multifractal analysis and found the transition to homogeneity occurring at length-scale $60 - 70 h^{-1}$ Mpc. Hogg et al. (2005) have analyzed the distribution of SDSS Luminous Red Galaxies (LRG) to find the transition to homogeneity at $\sim 70 h^{-1}$ Mpc. Sarkar et al. (2009) used the multifractal analysis of SDSS DR6 to find the transition to homogeneity at length-scale greater than $70 h^{-1}$ Mpc. Scrimgeour et al. (2012) find homogeneity at length-scale $70 - 80 h^{-1}$ Mpc in the redshift range $0.2 - 0.8$ using WiggleZ survey. Nadathur (2013) find homogeneity at length-scale above $130 h^{-1}$ Mpc for the SDSS DR7 quasar catalogue containing the Huge-LQG at redshift $z \sim 1.3$. Some recent studies with Shannon entropy show that the main galaxies from SDSS DR12 and luminous red galaxies (LRG) from SDSS DR7 are homogeneous beyond $150 h^{-1}$ Mpc (Pandey & Sarkar 2015; Pandey & Sarkar 2016).

As there is no consensus in the issue of cosmic homogeneity, it is important to test the assumption on multiple datasets with different statistical tools. If the assumption is ruled out with high statistical significance by multiple datasets there would be a major paradigm shift in cosmology.

SDSS is the largest and finest galaxy redshift survey todate which provides us an unique opportunity to test the assumption of cosmic homogeneity on very large scales with unprecedented confidence. SDSS provides distributions of different types of galaxies out to different distances covering enormous volumes. Galaxies are believed to be a biased tracer of the underlying mass distribution and differ-

ent types of galaxies are expected to have different values of linear bias. In the present work we would like to test the assumption of homogeneity using different tracers of the underlying mass distribution. We use the multifractal analysis (Martinez & Jones 1990; Coleman & Pietronero 1992; Borgani 1995) to characterize the scale of homogeneity in the main galaxy sample, LRG sample and Quasar sample from the Sloan Digital Sky Survey (SDSS) (York et al. 2000). We have considered the main galaxy sample (MGS) from SDSS DR7, Luminous Red Galaxy (LRG) sample from SDSS DR7 and Quasars sample from SDSS DR12. The LRG sample covers a much larger volume as compared to the main galaxy sample but it is only quasi volume limited and quite sparse as compared to the main sample. The quasar sample analyzed here is even larger in volume. It is the largest among these samples but also sparsest among them. These samples provide us the scope to test homogeneity upto different length scales due to their different volume coverages. Besides the different volumes and number densities, the samples also have different properties such as luminosity and colour resulting in different clustering properties. A combined analysis of the multiple tracers of the underlying mass distribution using the same statistical tools, provide us an unique opportunity to test the assumption of cosmic homogeneity for different tracers on a wide range of length scales.

A brief outline of the paper follows. In section 2 we describe the data followed by the method of analysis in section 3. We present the Results in section 4 and Conclusions in section 5.

2 DATA

The Sloan Digital Sky Survey (SDSS) (York et al. 2000) is a wide-field photometric and spectroscopic survey of sky using a 2.5 m Sloan Telescope at Apache Point Observatory in New Mexico, United States. It imaged the sky in five different pass-bands u, g, r, i and z . We have considered three different types of tracers namely the galaxies from the MAIN sample, the luminous red galaxies (LRGs) and the quasars from the SDSS. We have used $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$ and $h = 0.71$ throughout the analysis. The various data sets used in the present analysis are described below.

2.1 SDSS DR7 Main Galaxy sample

We have used the main galaxy sample from SDSS Data Release 7 (Abazajian et al. 2009) for which the target selection algorithm is described in Strauss et al. (2002). The main galaxy sample comprises of galaxies brighter than limiting r-band Petrosian magnitude 17.77. We have identified a contiguous region in the Northern Galactic cap which spans $-40^\circ \leq \lambda \leq 35^\circ$ and $-30^\circ \leq \eta \leq 30^\circ$, where λ and η are survey coordinates defined in Stoughton et al. (2002). We have constructed a volume limited sample of galaxies in the region by restricting the r-band absolute magnitude to the range $M_r \leq -20$. This absolute magnitude cut produces a volume limited sample of galaxies within $z \leq 0.106$. We finally extract 64109 galaxies in the redshift range of $0.040 \leq z \leq 0.106$ which corresponds to the radial comoving distances in the range $175.24 \leq r \leq 457.09 h^{-1}$ Mpc.

2.2 SDSS DR7 Luminous Red Galaxy sample

The Luminous Red Galaxy distribution (LRG) (Eisenstein et al. 2001) from the SDSS extends to a much deeper region of the Universe as compared to the SDSS Main galaxy sample as they can be observed to greater distances as compared to normal galaxies for a given magnitude limit. The LRGs have stable colors which make them relatively easy to pick out from the rest of the galaxies using the SDSS multi-band photometry. For the present analysis, we consider the spectroscopic sample of LRG extracted from the SDSS DR7. The LRG sample is quasi-volume limited to a redshift of $z = 0.36$. We use the DR7-Dim sample extracted by Kazin et al. (2010). This sample contains 67,567 galaxies within the redshift range of 0.16 to 0.36 for g-band absolute Magnitude (M_g) range brighter than -21.2 and fainter than -23.2 . We restrict our sample to a contiguous region in the Northern Galactic Cap which spans $-52^\circ \leq \lambda \leq 52^\circ$ and $-31^\circ \leq \eta \leq 31^\circ$ which constitutes our sample with 48,308 LRGs.

2.3 SDSS DR12 Quasars sample

The quasars are the brightest class of objects known in the Universe and their high luminosities allow us to detect them out to larger distances. We use the SDSS DR12 quasar catalogue for which the target selection is described in Ross et al. (2012) and the data is described in Pâris et al. (2014). The SDSS DR12 catalogue contain a total 297301 quasars. In the present work we use a quasar sample prepared by Sarkar & Pandey (2016). The preparation of the primary quasar sample is described in Sarkar & Pandey (2016). The primary quasar sample used in this analysis contain 117882 quasars with the g-band PSF magnitude $g \leq 22$ and the r-band PSF magnitude ≤ 21.85 (Ross et al. 2009). We select the quasars in a contiguous region $150^\circ \leq \alpha \leq 240^\circ$ and $0^\circ \leq \delta \leq 60^\circ$ where α and δ are the right ascension and declination respectively. The selected quasars have redshift in the range $2.2 \leq z \leq 3.2$. The resulting quasar sample have a varying number density in redshift. Constructing a strictly volume limited sample of quasars from this data results into a sample with a very poor number density. We construct a quasar sample with near uniform and reasonable number density in the entire redshift range by using a redshift dependent magnitude cut $M_i \geq M_{lim}(z)$ in the i-band absolute magnitude. The limiting magnitude $M_{lim}(z)$ is described by a polynomial of the form $M_{lim}(z) = az^3 + bz^2 + cz + d$ where a, b, c and d are the coefficients to be determined. We constrain a, b, c, d so as to have a $< 30\%$ fluctuations in the comoving number density around the mean density at all the redshifts in the chosen redshift range. We find that $a = 24.206, b = -194.325, c = 511.413, d = -467.422$ provides us with a quasar sample consisting of 24,213 quasars distributed across a volume of $1.83 \times 10^{10} h^{-1} \text{Mpc}^3$ with a mean number density of $1.32 \times 10^{-6} h^3 \text{Mpc}^{-3}$. The quasar sample has a linear extent of $759.32 h^{-1} \text{Mpc}$ in the radial direction.

2.4 LasDamas simulations

Large Suite of Dark Matter Simulations (LasDamas) (McBride et. al., in preparation) is a cosmological N-body

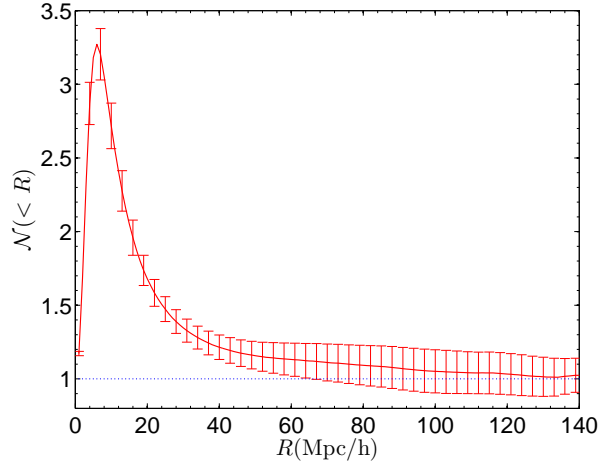


Figure 1. This plot shows the variation of scaled number counts $\mathcal{N}(<R)$ with radius R for the SDSS Main Galaxy sample. The $1 - \sigma$ errorbars are obtained from 30 Mock samples.

dark matter simulation project to obtain adequate resolutions in many large boxes rather than one single realization at high resolution. The cosmological parameters used in the LasDamas simulations are those of Λ CDM model: $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $\Omega_b = 0.04$, $h = 0.7$, $\sigma_8 = 0.8$ and $n_s = 1$. The galaxy mocks are generated by artificially placing galaxies inside dark matter halos using a halo occupation distribution with parameters fit from the respective SDSS galaxy samples. We have used the gamma release of the galaxy mock sample generated from Oriana simulations. The simulation was carried out in a simulation box of side $2400 h^{-1} \text{Mpc}$ with 1280^3 particles. The individual particles in the simulation has a mass of $45.73 \times 10^{10} h^{-1} \text{M}_\odot$ and the spatial resolution of the simulation is $53 h^{-1} \text{Kpc}$. We have used 30 and 40 independent realizations of mock galaxy catalogue from LasDamas Simulations for the SDSS MAIN and LRG sample respectively. We then extract mock samples for each of our SDSS MAIN sample and LRG sample which have the same geometry and number density as the actual data. The mocks were analyzed exactly in the same way as the SDSS data.

2.5 Random samples

We have generated random samples for each of our data sets. The random distributions have the same number density and they are distributed over the same region having identical geometry as the actual samples. In each case we generate 50 independent realizations of random samples. The random samples were analyzed in the same way as the actual samples.

3 METHOD OF ANALYSIS

We have used two statistical measures to study the transition to homogeneity. One is based on the simple number counts in spheres of radius R and its scaling with R . The other one is the Multifractal analysis which uses the scaling of different moments of the number counts.

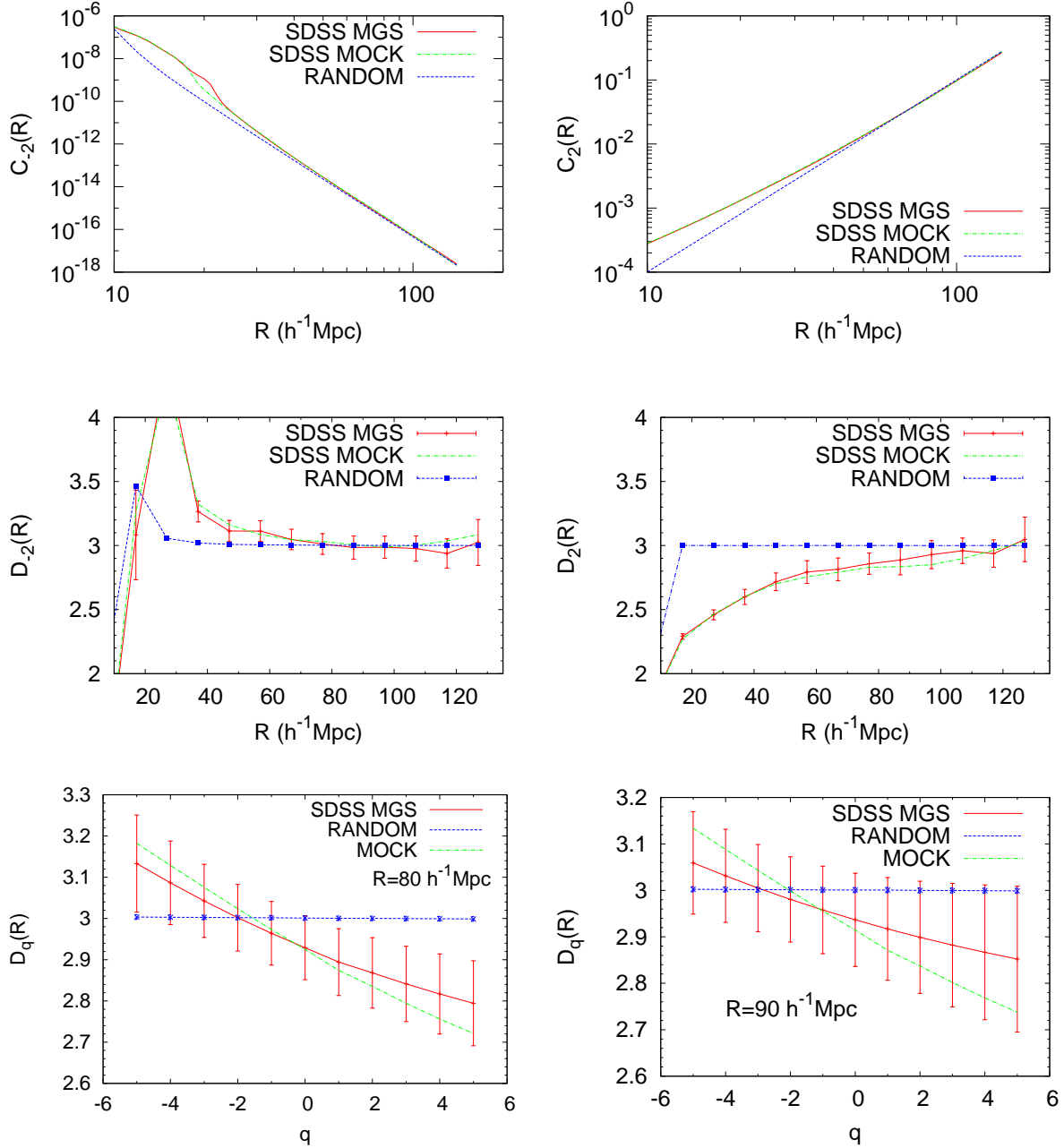


Figure 2. The top two panels show the variation of Correlation integral $C_q(R)$ with radius R for $q = 2$ and $q = -2$ for the SDSS main galaxy sample. The variation of $D_q(R)$ with respect to R for $q = 2$ and $q = -2$ for the same galaxy sample are shown in the middle two panels. The bottom two panels show the variation of D_q with q for two different values of R marking the transition to homogeneity. The $1 - \sigma$ error bars shown in different panels are obtained from the mock samples.

3.1 Scaled Number Counts

One of the simple test of homogeneity is to check how the number of points included inside spheres changes with the radius of the spheres. For this, we consider the galaxies as centres and place spheres of radius R around these centres to count the number of galaxy $N(< R)$ within these spheres. We consider only those galaxies as centres for which the spheres lie completely within the survey region. The number counts $N(< R)$ is expected to scale as

$$N(< R) \propto R^D \quad (1)$$

for a homogeneous distribution, where D is the ambient dimension. In three dimension we expect $D = 3$ at the scale of homogeneity. Random distributions are considered to be homogeneous by construction. We take the average over all the galaxies, to obtain the mean $N(< R)$ for a distribution and define an estimator,

$$\mathcal{N}(< R) = \frac{\sum_{i=1}^{N_c} \rho_i^2 \left(\frac{N_{Ai}(< R)}{N_{Ri}(< R)} \right)}{\sum_{i=1}^{N_c} \rho_i^2}, \quad (2)$$

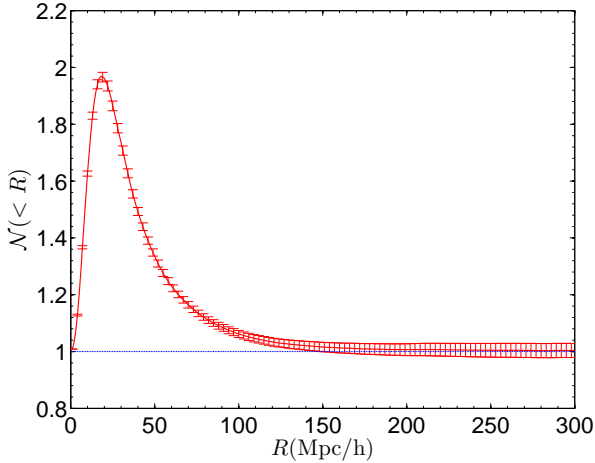


Figure 3. This plot shows the variation of scaled number counts $\mathcal{N}(<R)$ with radius R for the SDSS LRG sample. The $1 - \sigma$ errorbars are obtained from 40 Mock samples.

where $N_A(<R)$ and $N_{R_i}(<R)$ are the average number counts for SDSS galaxy sample and i^{th} random samples, ρ_i is the ratio of number of galaxies in the i^{th} random sample and SDSS sample and N_c is the number of random catalogues used in the analysis. It may be noted that the number of galaxies in the random sample and the SDSS sample may not be identical due to the exclusion of spheres near the survey boundary. We expect the value of $\mathcal{N}(<R)$ to be unity at the transition scale to homogeneity.

3.2 Multifractal Analysis

Galaxy surveys suggest that the Universe resembles a fractal on small scales. The fractal dimensions are commonly used to characterize fractals. There are various ways to measure the fractal dimension (Borgani 1995), out of which the box counting dimension and correlation dimensions are simple to compute in finite distributions. The Box counting dimension and the correlation dimension quantify different aspects of the scaling behaviour. These represent the particular cases of the generalized Minkowski-Bouligand dimensions D_q , where q represents different moments of galaxy counts. The value of D_q at $q = 1$ and $q = 2$ corresponds to Box counting dimension and Correlation dimension respectively. For a fractal distribution, the values of D_q will be different for different values of q where $q > 0$ gives more weightage to overdense region and $q < 0$ gives more weightage to the underdense region.

The generalized correlation integral is defined as,

$$C_q(R) = \frac{1}{MN} \sum_{i=1}^M [n_i(<R)]^{q-1}, \quad (3)$$

where $n_i(<R)$ is the number count within a sphere of co-moving radius R centered on the i^{th} galaxy, M is the number of centers chosen and N is the total number of galaxies.

The generalized Minkowski-Bouligand dimension is given by,

$$D_q(R) = \frac{1}{q-1} \frac{d \log C_q}{d \log r}, \quad (4)$$

We have considered q in the range $-4 \leq q \leq 4$. The finite number of galaxies restrict us to consider any arbitrary large values of $|q|$ (Bouchet et al. 1991). For the calculation of D_q from C_q , we have used the numerical differentiation by riddle's method describe in Press et al. (1992). The random samples are considered to be homogeneous by construction. So to determine the scale of transition to homogeneity, we compare the results for various galaxy samples with that from the random samples. If the galaxy sample is homogeneous then we expect the value of D_q to be consistent with that of random samples at the transition scale to homogeneity.

4 RESULTS

4.1 Results from the SDSS DR7 Main galaxy sample

Figure 1 shows the variation of the scaled Number counts $\mathcal{N}(<R)$ with R for the main galaxy sample of the SDSS. The $1 - \sigma$ error bars are estimated from 30 mock samples. We define the transition scale to homogeneity where the value of $\mathcal{N}(<R)$ is within 1% of its expected value 1. The result suggests that the MAIN galaxy sample is homogeneous beyond a length-scale of $\sim 70 - 80 h^{-1}$ Mpc.

We show the correlation integral $C_q(R)$ as a function of radius R for $q = -2$ and $q = 2$ in the top two panels of Figure 2. The solid line and the dot dashed line shows the results for the SDSS main galaxy sample and its mock counterparts from the LasDamas N-body simulations and they are nearly indistinguishable from each other. The result for the random samples is shown with the dashed line. We find that $C_q(R)$ increases with R for positive values of q whereas it falls progressively for negative values of q . It is quite clear from the plot that on large scales the main galaxy sample of SDSS, the corresponding mock samples and the random samples all exhibit the same scaling behaviour. But the scaling behaviours for the random samples are noticeably different from that of the SDSS main galaxy sample and the mock samples at $R < 40 h^{-1}$ Mpc. It may be noted that for both $q = -2$ and $q = 2$ these differences diminish with increasing length scales R . The behaviour of $C_q(R)$ is similar for other positive and negative values of q .

We show $D_q(R)$ as a function of R for $q = -2$ and $q = 2$ for the main galaxy sample in the middle two panels of Figure 2. The $1 - \sigma$ error bars for the SDSS data are obtained from 30 mock samples. We expect $D_q(R)$ to match with the ambient dimension for a homogeneous distribution. The random samples are homogeneous by construction and we expect them to have $D_q(R) = 3$ at all R . We can see in both the middle panels of Figure 2 that for the SDSS main galaxy sample and their mock samples $D_q(R)$ values converge to 3 within 1% at scales $\sim 70 - 80 h^{-1}$ Mpc. Here we adopt a working definition for the scale of homogeneity to be the scale where $D_q(R)$ comes within 1% of 3 (Scrimgeour et al. 2012) or 1% of the results of the random samples whichever earlier. It may be noted that D_q values for the SDSS and random samples differs greatly on small scales despite having the same mean inter particle separation. This clearly indicates that the SDSS main galaxy sample is homogeneous above a length scales of $80 h^{-1}$ Mpc.

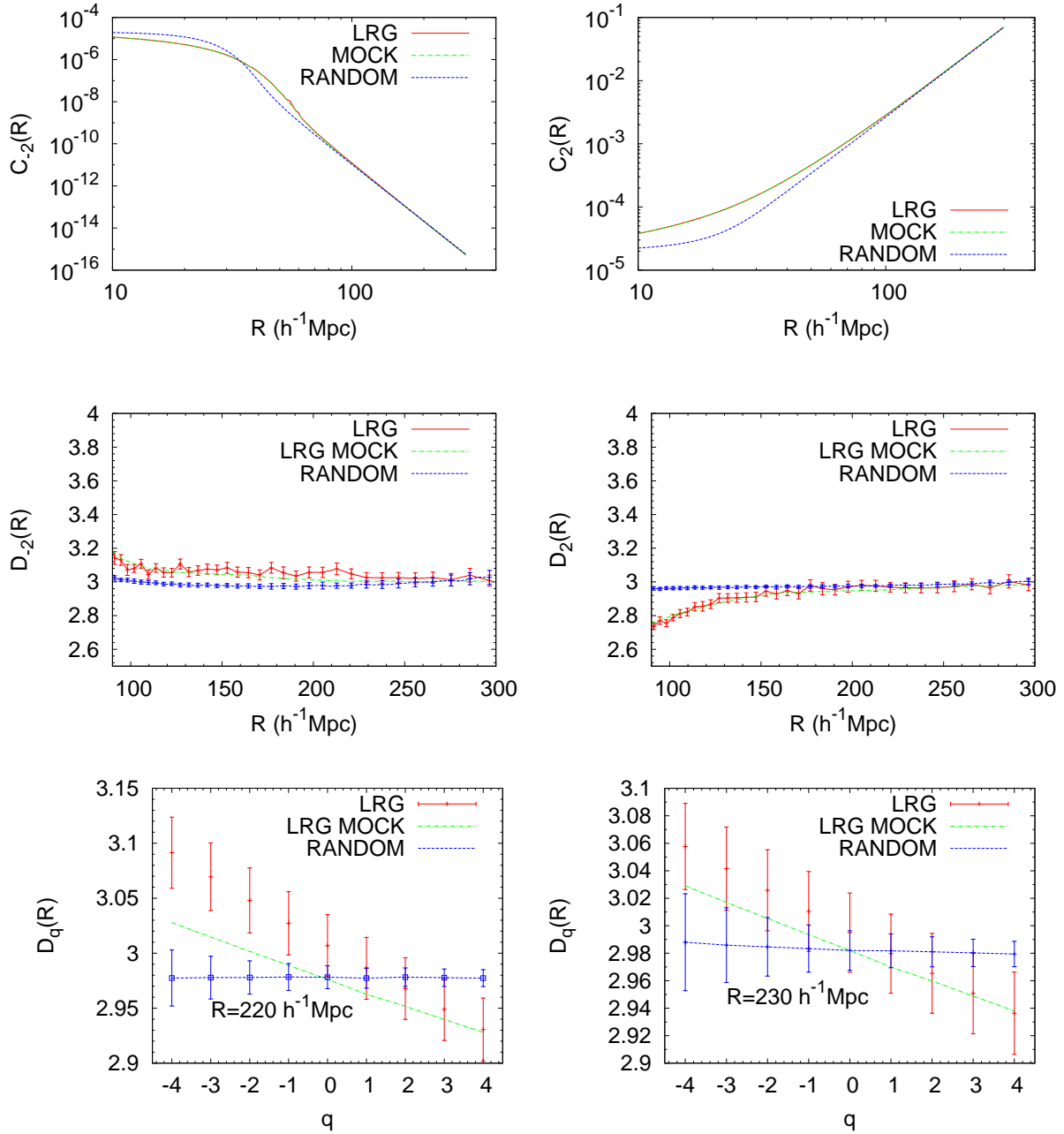


Figure 4. Same as Figure 2. but for the SDSS LRG sample.

In the left and right bottom panels of Figure 2, we show the variation of D_q across the different q values for $R = 80 h^{-1} \text{Mpc}$ and $R = 90 h^{-1} \text{Mpc}$. Clearly at $R = 80 h^{-1} \text{Mpc}$, the D_q values for the main galaxy sample and its mock counterparts show a large offset from that observed in the random samples for the entire range of positive q . Interestingly, increasing R to $90 h^{-1} \text{Mpc}$ reduces these offsets and the results for the random samples comes well within the $1 - \sigma$ errorbars of the results from the main galaxy sample. This indicates the presence of a transition scale to homogeneity in the SDSS main galaxy distribution at a length scale of $90 h^{-1} \text{Mpc}$.

4.2 Results from the SDSS DR7 LRG sample

Figure 3 shows the variation of measured Number counts $\mathcal{N}(< R)$ with R for the SDSS LRG sample. The $1 - \sigma$ error bars are estimated from 40 mock samples. Following the definition stated earlier, we find that the LRG sample becomes homogeneous at length-scale of $\sim 150 h^{-1} \text{Mpc}$.

The top two panels of Figure 4 show the variation of $C_q(< R)$ with respect to R for $q = -2$ and $q = 2$. We find that for $q > 0$, $C_q(< R)$ increases with R whereas for $q < 0$, it falls progressively. It is quite clear from the plots that the results from the mock samples are quite consistent with the LRG data on large scales. But their behaviours are quite different as compared to the random samples at $r < 80 h^{-1} \text{Mpc}$.

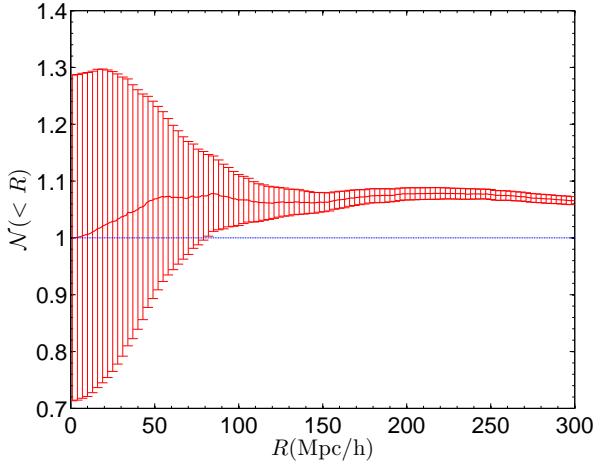


Figure 5. This plot shows the variation of scaled number counts $\mathcal{N}(<R)$ with radius R for the SDSS quasar sample. The $1 - \sigma$ errorbars are obtained from bootstrap resampling.

In the middle two panels of Figure 4 we show the variation of $D_q(R)$ with R for $q = -2$ and $q = 2$ for the SDSS LRG sample. The $1 - \sigma$ error bars for the LRG sample is estimated from 40 Mock samples. The $D_q(R)$ values for the LRG sample and its mock counterparts are quite different than the random samples at scale $< 230 h^{-1}$ Mpc for $q = -2$ and $< 150 h^{-1}$ Mpc for $q = 2$. Following the same working definition mentioned earlier we find that the $D_q(R)$ values for the LRG sample comes within 1% of 3 at a scale $\sim 200 h^{-1}$ Mpc.

The variation of D_q with q for two different values of R are shown in the bottom two panels of Figure 4. The bottom left panel in this figure corresponds to $R = 220 h^{-1}$ Mpc. This plot clearly shows the transition to homogeneity is reached for the positive values of q whereas the D_q for negative values of q still differs from the random samples indicating the non uniformity across the underdense regions at that scale. The right panel shows the same results but for $R = 230 h^{-1}$ Mpc. We find that the differences in the D_q values at negative q decreases noticeably and the errorbars for the LRG and random samples now overlap with each other. Clearly the spectrum of D_q values for the LRG and random data agree well when we change R from $220 h^{-1}$ Mpc to $230 h^{-1}$ Mpc. This indicates a transition scale to homogeneity in the LRG data at $\sim 230 h^{-1}$ Mpc.

4.3 Results from the SDSS DR12 Quasar sample

In Figure 5 we show the scaled number counts $\mathcal{N}(<R)$ as a function of R for the SDSS quasar sample. The $1 - \sigma$ error bars shown here are estimated from samples obtained by bootstrap resampling. We note that the scaled number counts for the quasar data does not show a distinct transition to homogeneity as seen earlier in the main galaxy sample and the LRG sample. The quasar distribution seems to be quite consistent with a homogeneous distribution from small scales upto a length scale of $\sim 70 h^{-1}$ Mpc. But the scaled number counts show a small and near constant deviation from its expected value 1 beyond $70 h^{-1}$ Mpc. This can not be emphasized due to a large mean inter-particle separation $\sim 91 h^{-1}$ Mpc of the SDSS quasar sample. This

departure does not necessarily imply that the quasar distribution is inhomogeneous and one needs to apply other statistical measures to asses its significance.

We show the variation of $C_q(<R)$ with R for the SDSS quasars for two different values of q in the top two panels of Figure 6. We find that for both $q = -2$ and $q = 2$, apparently the value of the correlation integral $C_q(<R)$ for the quasar sample is quite close to that for the random samples throughout the entire length scale with small differences.

In the middle left panel and right panels of Figure 6 we show the variation of D_q with R for the quasar sample for $q = -2$ and $q = 2$ respectively. We find that the D_q values for the quasar sample for $q = 2$ are quite consistent with that from a random distribution for $R > 100 h^{-1}$ Mpc whereas the D_q values differ from that a random distribution for $q = -2$ upto a length scales of $\sim 200 h^{-1}$ Mpc.

We show how $D_q(R)$ depend on q for the quasar sample for $R = 100 h^{-1}$ Mpc and $R = 150 h^{-1}$ Mpc in the bottom left and right panels of figure 6 respectively. Clearly at $R = 100 h^{-1}$ Mpc the transition to homogeneity is observed for the positive values of q whereas the D_q for negative values of q still differs from the random samples indicating the non uniformity across the underdense regions at that scale. We also note that the D_q values for both the quasar sample and the random samples deviate from 3 for almost all the values of q . This may not be surprising as this length scale is very close to the mean interparticle separation in the quasar and random samples. The right panel shows the same results but for $R = 150 h^{-1}$ Mpc. We find that the differences in the D_q values at negative q decreases noticeably and the results for the random samples are well within $1 - \sigma$ errorbars for the results from the quasars. Further at $R = 150 h^{-1}$ Mpc the D_q values converge to ~ 3 for positive q values whereas the deviations still persists at negative q values. Interestingly the deviations are of opposite sign for the quasar and the random samples. Roberts (2005) find that the generalized dimensions D_q for $q \leq 0$ are extremely sensitive to the regions of low density for a finite size data sets. This implies that one cannot tell the difference between empty space and space that should be filled in with very low probability. These differences dramatically affect the generalized dimensions D_q for the negative q values. Considering the very low density of the quasar sample we conclude that the SDSS quasar distribution indicates a transition to homogeneity at $\sim 150 h^{-1}$ Mpc.

5 CONCLUSIONS

We used the scaled number counts and different moments of the number counts in spheres to identify the transition scale to homogeneity in the SDSS main galaxy sample, LRG sample and quasar sample. The scaled number counts in the main galaxy sample and LRG sample show a transition to homogeneity on scales of $80 h^{-1}$ Mpc and $150 h^{-1}$ Mpc respectively whereas the scaled number counts in the quasar sample is consistent with homogeneity on small scales with a small and near constant deviation from it on larger scales. It may be noted here that the quasar sample is the largest but sparsest among all the samples we have analyzed. Further we could not define a strictly volume limited sample for the quasars as done for the main galaxy sample. The

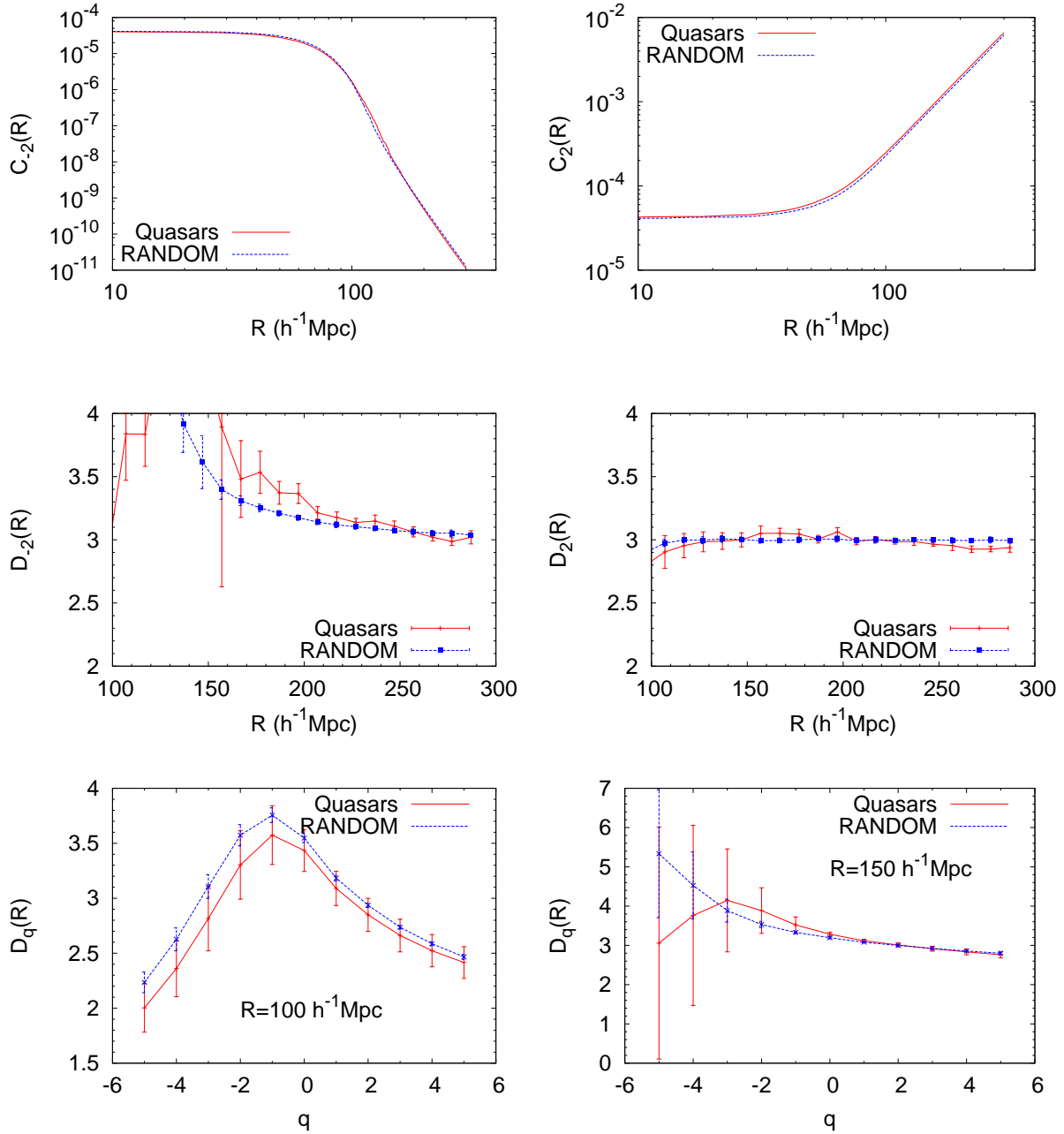


Figure 6. Same as Figure 2. but for the SDSS quasar sample.

LRG sample is a quasi volume limited in nature. To define our quasar sample we used a redshift dependent magnitude cut so as to maintain a near uniform comoving number density for them. This helps us to maintain a uniform comoving number density in the quasar sample but introduces a non-uniformity in the magnitudes of the quasars in the sample. Further the resulting quasar sample has a very poor number density with mean inter-particle separation of $\sim 91 h^{-1}$ Mpc and the number counts are expected to be dominated by shot noise below this length scale. Given the incompleteness of the quasar sample it is hard to interpret the behaviour of the scaled number counts in the quasar sample in a simple manner.

We analyzed the spectrum of the generalized dimension

for all three samples to find that the generalized dimension D_q show a transition to homogeneity at $90 h^{-1}$ Mpc, $230 h^{-1}$ Mpc and $150 h^{-1}$ Mpc for the SDSS main galaxy sample, the LRG sample and the quasar sample respectively. These values are somewhat different than that obtained with the scaled number counts of the respective samples. This may arise due to higher sensitivity of D_q to underdense regions. One may note that these samples have quite different number densities. The number density decreases by order of magnitudes as we go from main galaxy sample to the LRG sample and the quasar sample. The transition scale to homogeneity are particularly different for the LRG and the quasar sample when they are determined from the q dependence of the generalized dimension D_q instead of the

behaviour of their scaled number counts. It is thus important to note that the transition scale to homogeneity is also sensitive to the statistical measures employed.

The present analysis indicates that there exist a transition scale to homogeneity for all the tracers of the mass distribution used in this analysis. We find the transition scale to homogeneity to be quite different for the normal galaxies, LRGs and quasars. This may originate from the fact that these distributions are biased tracers of the underlying mass distribution and the large scale linear bias for these samples are expected to be quite different from each other. The quasars are found to inhabit dark matter halos of constant mass $\sim 2 \times 10^{12} h^{-1} M_{\odot}$ from the time of peak quasar activity ($z \sim 2.5$) and their large scale linear bias evolves from $b = 3$ at $z \sim 2.2$ to $b = 1.38$ at $z \sim 0.5$ (Shen et al. 2013; Ross et al. 2009; Geach et al. 2013). Our quasar sample extends from $z = 2.2$ to $z = 3.2$ for which we expect a large scale linear bias of > 3 . On the other hand the SDSS LRG sample is known to have a large scale linear bias values of $b \sim 2$ (Marín 2011; Sawangwit et al. 2011). So one would expect the quasar sample to be homogeneous on larger scales than the LRG sample. Sarkar & Pandey (2016) applied an information theory based method (Pandey 2013) to analyze the SDSS DR12 quasar sample and find that the quasar distribution is homogeneous beyond a length scale of $250 h^{-1}$ Mpc. Information entropy is related to the higher order moments of a distribution and may be more sensitive to the homogeneities present in a distribution. But the present analysis shows that the SDSS quasar sample is homogeneous on a length scale which is smaller than this and also smaller than the transition scale to homogeneity for the LRGs. This indicates that the clustering strength as indicated by their bias values may not be the only parameter deciding the transition scale to homogeneity. It should be also noted here that the quasar sample is very sparse. The poor number density of the quasar sample together with its somewhat incomplete nature may also yield such differences in our results. However despite these differences all the galaxy samples from the SDSS including the main sample, the LRG sample and the quasar sample exhibit a transition to homogeneity on large scales. The present analysis thus validates the fundamental assumption of cosmic homogeneity and reaffirms that the Universe is indeed statistically homogeneous on sufficiently large scales irrespective of the tracers.

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